

MATH 141 Sample Exam 1

Question 1 Determine the following limits (if they exist):

a) $\lim_{x \rightarrow -2} \frac{x - |x^3 - 4|}{|x| + 3}$

b) $\lim_{x \rightarrow 4} \frac{x^2 - 16}{\sqrt{x} - 2}$

c) $\lim_{x \rightarrow 4^-} \frac{\sqrt{x+5} - 4}{x^2 - 2x - 8}$

d) $\lim_{x \rightarrow 0} \frac{\sin 2x}{4x}$

e) $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{2x^2}$

f) $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$

Question 2 Find a value of A so that the function $f(x)$ is continuous for all values of x .

$$f(x) = \begin{cases} \cos(\pi x) + \sqrt{4-x} & \text{if } x \leq 2 \\ \sqrt{2} + \sin(A\pi x) & \text{if } x > 2 \end{cases}$$

Question 3 Show that the equation $3^x = 2 - x^2$ has at least one solution on the interval $[0, 1]$.

Question 4 Prove that $\lim_{x \rightarrow -2} 3x + 7 = 1$.

Question 5 Prove that $\lim_{x \rightarrow 3} x^2 - 1 = 8$

Question 6

a) Use the (limit) definition of derivative to compute the derivative $f'(x)$ for the function

$$f(x) = \frac{3x - 1}{2x + 3}.$$

b) Find the equation of the tangent line to $y = \frac{3x - 1}{2x + 3}$ at the point $x = 3$.

Question 7 Compute the derivatives of the following functions (do not simplify):

a) $f(x) = 4x^3 - \sqrt[3]{x}$

b) $f(x) = \left(3x^2 - \frac{4}{x^2}\right) (3x^7 - 8x^4 + 2x^{-1/4})$

c) $f(x) = \frac{x^2 - \sqrt{x}}{5x + 3x^{-2}}$

d) $f(x) = (3x^3 - 4x + 9)^{7/3}$

e) $f(x) = \cos^7 \left(4x^2 - \frac{5}{\sqrt{x}}\right)$

Question 8 Find the value(s) of x where the graph of $y = 3x^{2/3} + 4x$ has a horizontal tangent line.

Question 9 For the function $g(t) = 4 - 3t + t^3$. Find the average rate of change of $g(t)$ over the interval $[-2, 3]$. What is the instantaneous rate of change of $g(t)$ at $t = 1$?